EXECUTION ACCURACY OF THE TENON JOINT FIT ESTABLISHED ON THE BASIS OF THE DIMENSION SCATTER IN SERIES PRODUCTION

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Abstract. The fitting of the tenon joint plays an important role in its functioning. Experiments were carried out in industrial conditions with the aim to investigate the execution accuracy of the tenon joint tolerated in two directions. On the basis of measurements and calculations of a series of tenon joints manufactured from two kinds of timber, namely alder and pine, the authors determined types and classes of fits of seat lengths with tenon widths and – for the same elements – of seat widths with tenon thicknesses.

Key words: seat, tenon, tolerance, fitting

INTRODUCTION AND RESEARCH OBJECTIVE

The discussed problem deals with the determination of the type and class of the tenon joint fit established on the basis of knowledge of the dimensional variability interval observed in the assembled elements.

It is evident from the literature on the subject [Szydlowski 1981] that the situation in which a dimension, which is a random variable undergoing normal distribution, deviates from its mean value \( \bar{x} \) by more than a threefold value of the standard deviation \( \sigma \) is very highly unlikely and, on average, can occur three times per one thousand. Therefore, its practical limits of variability are assumed as \( \bar{x} + 3\sigma \) and \( \bar{x} + 3\sigma \) [Greber 1997].

There are very few references in the literature connected with wood industry dealing with the true fit of the tenon joint occurring in industrial conditions.

The research results concerning optimal clearances in joints made in laboratory conditions [Rybski 1976] have very little to do with the clearance values occurring in real industrial conditions in which the type of fit depends on the class of the execution accuracy of seats and tenons.
If we assume that tolerating is a deliberate limitation of the acceptable dimension variability, then a question might be asked: what is the true execution accuracy of the tenon joint fit? In the case of this type of joint, the achievement of the assumed fit encounters a number of difficulties resulting from the processing specificity and material properties [Bieliński and Korzeniowski 1979]. It is not possible to assume in advance the type of fit of a tenon joint without the dimensional assessment of its execution accuracy because clearances and negative allowances in joints depend on specific conditions occurring in the course of processing operations in industrial practice. The knowledge of the fit in a real joint allows such mutual positioning of tolerance fields in the elements to be assembled in future that the occurring clearances and negative allowances approach the optimal ones as closely as possible [Jezierski 1983].

The aim of the experiment was to determine, on the basis of measurements and statistical calculations of assembled elements manufactured in industrial conditions, the fit type and class of a tenon joint tolerated in two directions. In addition, the study aims to disseminate procedures which would allow determining in industrial conditions the type and class of fit in the tenon joint.

The proposed procedures should allow workers employed in the wood industry to assess the dimensional accuracy of the execution of elements making the joint with regard to their meeting the assumed type of fit [Bajkowski 1990, Staniszewska and Zakrzewski 1988, 1990, Zakrzewski and Staniszewska 2002].

METHODOLOGICAL ASSUMPTIONS

Experimental material

The objects of fitting were tenon joints of chairs and wardrobes of the furniture set “Jantar” manufactured in a furniture factory in Pniewy. The employed subunits are manufactured from two timber species, namely alder and pine. The moisture content of the fitted elements was measured with the assistance of a WRD-100 type hygrometer with a digital display of the Tanel Company. The average moisture content of the experimental material amounted to 10% at the air temperature of 21°C.

The method employed to take measurements

The seats occurring in joints were made with a horizontal, single-spindle, oscillating drilling and milling machine of the Balestrini Company using a four-blade shank cutter of 8 mm nominal diameter and rotational velocity of about 4500 min\(^{-1}\).

Tenons were made with a Bacci Company circular tenoning machine using a saw with 180 mm diameter and a cutter with 90 mm diameter. The rotational velocity of the spindle was about 3500 min\(^{-1}\).

Figure 1 shows the tenon joint with dimensions tolerated in two directions. The \textbf{a} direction refers to the seat length and tenon width fit, whereas the \textbf{b} direction – to the fit of the seat width with the tenon thickness.

Measurements of the \textbf{a} and \textbf{b} directions were taken with a Tesa Company electronic calliper. Each dimension was measured three times in three measuring fields across the
Execution accuracy of the tenon joint fit established...

seat width and the width and thickness of the tenon. On the other hand, the seat length was measured in one measuring field until its maximum value was obtained.

Two twenty-element samples were accepted for measurements which were selected on the basis of random choice as specified in the PN-83/N-03010 standard “Statistical quality control”.

Statistical calculation of the fit accuracy execution

It is impossible to manufacture a series of elements of ideal dimensional accuracy. Assumptions concerning the occurrence only of random errors cause normal distribution [Andrzejewski et al. 1993]. In practice, they are not always fulfilled strictly because randomness is systematically weakened by various factors resulting in the development of deviations from the normal distribution depending on the direction and intensity of these factors. Most frequently, we can only talk about the greater or smaller approximation of actually obtained distribution of results to that of the normal distribution [Czyżewski 1993].

As it is well known, the scatter parameters include: the arithmetic mean of dimensions \( \bar{x} \) and the standard deviation \( \sigma \). This deviation is a measure of scatter of the observed dimensions.

In metrology, the variability interval of the random variable, such as the examined dimension, is traditionally determined by the value of six standard deviations \( \sigma \), i.e. \( T_r = 6\sigma \). This interval is contained between the following two values: the upper \( A_r = \bar{x} + 3\sigma \) and the lower \( B_r = \bar{x} - 3\sigma \) observed dimensions. The fitted elements possess the same basic dimension of the fit \( N_r \) which is treated as the initial dimension in relation to which deviations are calculated:

- the observed upper deviation of the \( ES_r \) hole which is the difference between the observed upper dimension of the \( B_r \) hole and the basic dimension \( N_r \),

Fig. 1. Graphic illustration of the fit: a – seat length with the tenon width, b – seat width with the tenon thickness

Rys. 1. Ilustracja graficzna pasowania: a – długość gniazda z szerokością czopa, b – szerokość gniazda z grubością czopa

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– the observed lower deviation of the $EI_r$ hole which is the difference between the observed lower dimension of the $A_{ro}$ hole and the basic dimension $N_r$,
– the observed upper deviation of the $es_r$ cylinder which is the difference between the observed upper dimension of the $B_{rw}$ cylinder and the basic dimension $N_r$,
– the observed lower deviation of the $ei_r$ cylinder which is the difference between the observed lower dimension of the $A_{rw}$ cylinder and the basic dimension $N_r$.

The mutual dimensional interrelation of the assembled elements follows from the adopted fitting principle, i.e. of the constant hole (BN-81/7140-11).

The kind of fitting between a specific seat and the tenon to be connected with it depends on the value of the clearance $L_r$ which occurs in this joint. The above value depends on the difference between the observed dimensions of the seat and tenon. The maximum clearance is calculated from the following formula:

$$L_{r,\text{max}} = B_{ro} - A_{rw} = ES_r - ei_r$$

while the minimum clearance, from

$$L_{r,\text{min}} = A_{ro} - B_{rw} = EI_r - es_r = -es_r$$

because in the principle of the constant hole, $EI_r = 0$, hence $N_r = A_{ro}$.

The interval of clearance variability is the difference between the maximum and minimum clearance

$$T_{pr} = L_{r,\text{max}} - L_{r,\text{min}}$$

which equals to the sum of the hole and cylinder variability intervals.

The point of departure for the determination of the kind and class of the fit is the comparison of the variability interval of dimension and tolerance. This can be achieved easiest by comparing deviations calculated for the fit with deviations contained in the appropriate standard. The observed deviations (designated with index $r$) should not exceed acceptable deviations because in this case the dimensional variability intervals of the assemble elements are contained within their tolerances.

The type and class of the fit were selected on the basis of the comparison of the observed deviations and those found in the standard:

$$ES_r \leq ES$$

$$es_r \leq es$$

$$ei_r \geq ei$$

where: $ES$, $es$ and $ei$ – deviations contained in the standard.

**RESEARCH RESULTS AND THEIR ANALYSIS**

Table 1 presents the results of joint measurements and statistical calculations of the seat length with the tenon width for the nominal setting of machine tools. This is the direction of tolerancing which in Figure 1 is designated with letter $a$. The designed nominal dimension of the joint was 30.00 mm. Measurements were carried out separately for the alder and pine joints.
It is evident from Table 1 that each time, the basic fit dimension (on the basis of the analysis of the observed dimensions), is smaller than the nominal (designed) dimension.

<table>
<thead>
<tr>
<th>Value Wielkość</th>
<th>Designation Oznaczenie</th>
<th>Alder Olsza</th>
<th>Pine Sosna</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arithmetic mean</td>
<td>$\bar{x}$</td>
<td>30.11</td>
<td>30.06</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>$\sigma$</td>
<td>0.31</td>
<td>0.26</td>
</tr>
<tr>
<td>Dimension scatter</td>
<td>$T_r$</td>
<td>1.86</td>
<td>1.56</td>
</tr>
<tr>
<td>Observed upper dimension</td>
<td>$B_u$</td>
<td>31.04</td>
<td>30.84</td>
</tr>
<tr>
<td>Observed lower dimension</td>
<td>$A_l$</td>
<td>29.18</td>
<td>29.28</td>
</tr>
<tr>
<td>Basic fit dimension</td>
<td>$A_{\text{det}} = N_r$</td>
<td>29.18</td>
<td>29.77</td>
</tr>
<tr>
<td>Upper deviation</td>
<td>$ES_r, es_r$</td>
<td>1.86</td>
<td>1.66</td>
</tr>
<tr>
<td>Lower deviation</td>
<td>$EI_r, ei_r$</td>
<td>0</td>
<td>0.10</td>
</tr>
<tr>
<td>Maximal clearance</td>
<td>$L_{c_{\text{max}}}$</td>
<td>1.76</td>
<td>1.32</td>
</tr>
<tr>
<td>Minimal clearance</td>
<td>$L_{c_{\text{min}}}$</td>
<td>-1.66</td>
<td>-0.90</td>
</tr>
<tr>
<td>Mean clearance</td>
<td>$L_{c_{\text{m}}}$</td>
<td>0.05</td>
<td>0.21</td>
</tr>
<tr>
<td>Interval of clearance variability</td>
<td>$T_{cr}$</td>
<td>3.42</td>
<td>2.22</td>
</tr>
<tr>
<td>Kind of fit</td>
<td>Rodzaj pasowania</td>
<td>H9 l9</td>
<td>H7 k8</td>
</tr>
<tr>
<td>Tolerated dimension</td>
<td>Wymiar tolerowany</td>
<td>29.18$^{+0.03}_{-0.00}$</td>
<td>29.77$^{+0.13}_{-0.37}$</td>
</tr>
</tbody>
</table>

The scatter measure of the measurement results as represented by the standard deviation is different for the seat and tenon. In the case of the seat from alder wood, it amounts to 0.31 mm, while for the tenon – 0.26 mm; for pinewood the appropriate values are: 0.16 mm and 0.21 mm, respectively. Therefore, the clearance scatter for the fit of elements manufactured from alder was 3.42 mm, while that from pine – 2.22 mm.

The execution accuracy of the fit was determined by comparing the observed variability intervals of the seat and tenon with the appropriate tolerances of the seat and
tenon dimensions contained in the BN-81/7140-11 standard. Graphic illustrations of the calculated variability intervals of the seat length, tenon width and the clearance are presented in Figure 2 for alder wood and in Figure 3 – for pinewood.

Fig. 2. Graphic illustration of the dimensional variability interval for alder wood: a) seat, b) tenon, c) clearances

Fig. 3. Graphic illustration of the dimensional variability interval for pine wood: a) seat, b) tenon, c) clearances
Because the scatter of dimensions and fit clearances are extremely small in comparison with the nominal dimensions of the assembled seats and tenons, the graphic pictures present large magnifications of the observed dimensional variability without drawing the seat and tenon and drawing only the so called ‘zero line’ [Staniszweska and Zakrzewski 2002]. The zero line is a straight line which coincides with the line of the basic dimension (lower dimension of the hole, i.e. \( A_r = N_r \)) in relation to which the position of variability fields of the seat, tenon and clearance is determined algebraically and graphically. If the observed dimensional variability intervals of the seat and tenon coincided with those of the acceptable ones, than the appropriate conformity of the deviations \( ES_r = ES, es_r = es \) and \( ei_r = ei \) would also exist. In this situation, it would be possible to determine unequivocally the type of fit. However it is not so in reality. For example, on the basis of Table 1, the basic fit dimension together with the observed deviations can be written down as 29.77 \( ^{0.96}_0 \) for the seat (pinewood) and as 29.77 \( ^{0.90}_{-0.36} \) for the tenon.

The observed values of clearances were calculated as follows:

\[
L_{r \max} = B_{ro} - A_{rw} = 30.73 - 29.41 = 1.32 \text{ mm}
\]

\[
L_{r \min} = A_{ro} - B_{rw} = 29.77 - 30.67 = -0.90 \text{ mm}
\]

The clearance variability interval is the difference between the observed values of maximum and minimum clearances

\[
T_{pr} = L_{r \max} - L_{r \min} = 1.32 - (-0.90) = 2.22 \text{ mm}
\]

It equals to the sum of the dimensional variability intervals of the seat and the tenon

\[
T_{pr} = T_{ro} + T_{rw} = 0.96 + 1.26 = 2.22 \text{ mm}
\]

The comparison of the observed variability intervals with the seat and tenon deviations according to the BN-81/7140-11, the seventh class of execution accuracy of the seat of the acceptable deviation of \( ES = T_o = 1.13 \text{ mm} \) should be adopted for the basic dimension of 29.77 mm. This means that the seat length was manufactured more accurately (\( T_{ro} = 0.96 \text{ mm} \)) than it is specified by the appropriate accuracy class and the record of the seat length dimension is 29.77 \( ^{0.13}_{-0.06} \). It is evident from the comparison of deviations that \( 0.96 < ES < 1.13 \text{ mm} \). The accuracy class for the tenon width was selected similarly. It is evident from the observed record of the tenon width with deviations 29.77 \( ^{0.90}_{-0.36} \) that the fitting has the character of a slightly negative allowance (type “k”) and is characterised by the eighth accuracy class because:

\[
0.90 \text{ mm} = es, < es = 1.12 \text{ mm} \quad \text{and} \quad -0.36 \text{ mm} = ei, > ei = -0.37 \text{ mm}
\]

The numerical representation of the tenon width dimension will, therefore, amount to 29.77 \( ^{0.12}_{-0.37} \).

The standardized clearances are obtained from the following calculations:

\[
L_{\max} = B_o - A_w = ES - ei = 1.13 - (-0.37) = 1.50 \text{ mm}
\]

\[
L_{\min} = A_o - B_w = -es = 1.12 \text{ mm}
\]
and the fit tolerance:

\[ T_p = L_{\text{max}} - L_{\text{min}} = 1.50 - (-1.12) = 2.62 \text{ mm} \]

It is evident from the comparison that the clearances that the observed clearances are contained within standard clearances

\[ L_{r, \text{max}} = 1.32 < 1.50 = L_{\text{max}} \]
\[ L_{r, \text{min}} = -0.90 > -1.12 = L_{\text{min}} \]

The fit selected from the standard for the pine wood H7k8 (constitutes the seventh class of execution accuracy of the seat length and the eighth class of execution accuracy of the tenon width) and signifies a slightly negative allowance fit (the “k” symbol).

The fitting standards for the alder wood were selected similarly. Symbolically, they are represented as H9 l9 which indicates the ninth class of execution accuracy for both the seat length and tenon width. The “l” symbol signifies the negative allowance fit.

At the same time, on the basis of measurements, for the same elements of the joint, the authors determined the type and class of the fit for the seat width dimensions and tenon thickness. The results of measurements and calculations are presented in Table 2.

The numerical values of these dimensions (tolerancing direction \( b \)) are considerably smaller than those found in Table 1 (tolerancing direction \( a \)) and, hence, the execution tolerances also decrease. The designed nominal dimension for the tolerancing direction \( a \) was 8.00 mm.

It is evident from Table 2 that the observed dimensional variability intervals are different for the seat and the tenon and, therefore, most frequently the assembled elements differ with regard to their execution accuracy classes. Both alder and pine fitted elements were characterized by the same execution accuracy of the seat width because the dimensional variability interval amounted to 0.24 mm which, according to BN-81/7140-11, corresponds to the third class of accuracy. This standard recommends the fourth class of the execution accuracy for the tenon width; therefore, the actual execution was more precise than the recommendation. On the other hand, the variability interval for the tenon thickness was 0.36 mm for alder wood and 0.30 mm for pine.

It is evident from the comparison of the observed deviations of the dimension of the seat thickness with the deviations contained in the standard that they were manufactured with the accuracy of class five in the case of alder wood and class four in the case of pine wood. The symbol designation of the \( b \) dimension fit selected from the standard of elements manufactured from alder wood takes the form of H3h5, while for those made from pinewood – H3h4. This means that the fits carried out for both wood types were of the same, ‘sliding’ character, (symbol “h”).

The fitting of dimensions of the seat width and the tenon thickness (dimension \( b \)) when compared with the fitting of dimensions of the seat length and the tenon width (dimension \( a \)) is characterized by a considerably smaller clearance variability interval. It amounted to 0.60 mm for alder wood and 0.54 for pinewood, whereas for the fitting of the seat length with the tenon width, it amounted to 3.42 mm for alder wood and 2.22 mm for pinewood.

It is clear from the comparison of the \( a \) and \( b \) dimensions of fitting that the seat length and the tenon width are of the mixed fit character of the “l” type for alder and “k” for pine, whereas the seat width combined with the tenon thickness possesses the character of the movable fit of type “h”, both for alder and pine woods.
The strongest tenon joints exposed to changing loads which occur in chair elements are those for the maximum clearance 0.7 mm in the a fit direction and the slight clearance ranging from 0.0 to 0.1 mm in the b fit direction [Rybski 1976].

The mean clearance found for the a fit direction is positive and amounts to 0.05 mm for alder wood and 0.21 mm for pinewood. The respective values for the b fit direction are: 0.33 mm for alder wood and 0.28 mm for pinewood. Therefore, from the point of view of the maximum joint strength, the fits for both directions a and b show excessive values of clearances.
The variability intervals of $T_p$ clearances for the a fit direction are too big in comparison with the H4/4 fit recommended in the standard and, therefore, attempts should be made to reduce them. The observed fits H9/9 and H7k8 are characterized by accuracy smaller that H4/4 which anticipates the fit tolerance of $T_p = 1.04$ mm. In order to diminish the unfavourable clearance variability interval for the a fit direction, it would be necessary – in the first place – to analyze the condition of the machine tools and tools employed in the processing.

On the other hand, the clearance variability intervals for the b fit direction yield a much higher fit accuracy and the H3h5 for alder wood differs only slightly from the standard, while the H3b4 falls within the standard. The mean clearances can be brought to optimal values by the appropriate regulation of the machine tool settings of the tenon mean arithmetic width and thickness [Zakrzewski and Staniszewska 2002]. If the optimal clearance value for the b fit direction was accepted as $L_{w} = 0.05$ mm (after Rybski), then it would be necessary to increase the tenon thickness by changing the setting of the tenoning machine in accordance with the following formula: $\Delta x_{w} = L_{w} - L_{w}$. In the case of the alder wood, the change of setting of the tenon thickness would amount to: $\Delta x_{w} = 0.33 - 0.05 = 0.28$ mm, whereas for the pinewood $\Delta x_{w} = 0.28 - 0.05 = 0.23$ mm.

SUMMING UP

1. The type and class of the tenon joint fit accuracy tolerated in two directions was determined on the basis of dimension analysis of the assembled elements according to the principle that the observed dimensional variability interval is contained within the interval of their acceptable variability and fulfills the condition of minimal differences.

2. The clearance scatter resulting from measurements and statistical calculations of the assembled elements after their manufacture amounts for the dimensions of:
   – the seat length and tenon width manufactured from alder wood $T_{pr} = 3.42$ mm, whereas for pinewood $T_{pr} = 2.22$ mm,
   – the seat width and tenon thickness made from alder wood $T_{pr} = 0.60$ mm and for pinewood $T_{pr} = 0.54$ mm.

3. For the a fit direction, the ninth class of the seat and tenon execution accuracy for the negative allowance fit “I” was obtained for elements manufactured from alder wood, whereas in the case of pinewood, the seat was made in class seven and the tenon – in class eight of the slightly negative allowance fit “k”.

4. In the case of the b fit direction, class three of the execution accuracy was obtained for seats manufactured from both alder and pine woods, whereas the fit classes of tenons were as follows: alder wood – class five, pinewood – class four.

REFERENCES